Numerical Analysis (10th Edition)

Problem

Repeat Exercise 1 using Müller's method.

Reference: Exercise 1

Find the approximations to within 10^{-4} to all the real zeros of the following polynomials using Newton's method. a. f (x) = $x^3 - 2x^2 - 5$

b. f (x) = $x^3 + 3x^2 - 1$

c. f (x) = $x^3 - x - 1$

d. f (x) = $x^4 + 2x^2 - x - 3$

e. f (x) = $x^3 + 4.001x^2 + 4.002x + 1.101$ f.

f (x) = $x^5 - x^4 + 2x^3 - 3x^2 + x - 4$

Step-by-step solution

Step 1 of 93

The relevant theory to solve the given problem is provided below.

Muller's Method: It is similar to that of secant method. But, the secant method uses a line through two points on the curve to approximate the root. Muller's method uses a parabola passing through three points on the curve for the approximation.

Comment

Step 2 of 93

The derivation of Muller's method begins by considering the quadratic polynomial called the parabola of the form $P(x)=a(x-p_2)^2+b(x-p_2)+c$ that passes through the points $(p_0,f(p_0)),(p_1,f(p_1))$, and $(p_2,f(p_2))$ respectively. The constants $a,b,$ and c are obtained by applying the following conditions.

$$\left\{\begin{array}{l} f(p_0)=a(p_0-p_2)^2+b(p_0-p_2)+c, \\ f(p_1)=a(p_1-p_2)^2+b(p_1-p_2)+c, \text{ and} \\ f(p_2)=c \end{array}\right\}$$

Comment

Step 3 of 93

By solving the equations the following situation arises

$$\left\{\begin{array}{l} a=\dfrac{(p_0-p_2)[f(p_1)-f(p_2)]+(p_2-p_1)[f(p_0)-f(p_2)]}{(p_0-p_1)(p_1-p_2)(p_2-p_0)}, \\ b=\dfrac{(p_1-p_2)^2[f(p_0)-f(p_2)]-(p_2-p_0)^2[f(p_1)-f(p_2)]}{(p_0-p_1)(p_1-p_2)(p_2-p_0)}, \text{ and} \\ c=f(p_2) \end{array}\right\}$$

To determine the next approximation p_3 the quadratic formula

$$p_3=p_2-\left\{\frac{2c}{b+\operatorname{sgn}(b)\sqrt{b^2-4ac}}\right\} \text{is applied}$$

Comment

Step 4 of 93

Once the approximation p_3 is found, the procedure is reinitialized by using $p_1, p_2,$ and p_3 to determine the next approximation p_4 and the method is continued until a satisfactory approximation is obtained.

At each step, Muller's method involves the radical $\sqrt{b^2-4ac}$, so the method yields approximate complex roots when $\sqrt{b^2-4ac}<0$

Comment

Step 5 of 93

a. Find the approximations to within 10^{-4} to all the real zeros of the polynomial $f(x)=x^3-2x^2-5$ Using Muller's method by the steps as shown below.

Step1: Find the interval $[a,b]$ in which the required real zero lies by using Maple Technology as shown below.

$> f := x^3 - 2\cdot x^2 - 5$

$f := x^3 - 2x^2 - 5$

$> a := \operatorname{convert}\left(f\big|_{x=2}, \mathit{float}, 8\right)$

$a := -5.$

$> b := \operatorname{convert}\left(f\big|_{x=3}, \mathit{float}, 8\right)$

$b := 4.$

Thus, the required interval is given by $[2,3]$

Comment

Step 6 of 93

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology as shown below.

$> f := x \to x^3 - 2\cdot x - 5$

$f := x \to x^3 - 2x - 5$

$> p0 := 0;\ p1 := 1;\ p2 := 2$

$p0 := 0$

$p1 := 1$

$p2 := 2$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := -5$

$f1 := -6$

$f2 := -1$

Continuation of the above

$> c := f2$

$c := -1$

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 3$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 8$

$> p3 := \operatorname{evalf}\left(p2 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := 2.119632981$

Comment

Step 7 of 93

Continuation of the above

$> p0 := 1;\ p1 := 2;\ p2 := p3$

$p0 := 1$

$p1 := 2$

$p2 := 2.119632981$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := -6$

$f1 := -1$

$f2 := 0.283914304$

$> c := f2$

$c := 0.283914304$

Comment

Step 8 of 93

Continuation of the above

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 5.119632984$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 11.34458689$

$> p4 := p3 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := 2.094317350$

Comment

Step 9 of 93

Continuation of the above

$> p0 := 2;\ p1 := 2.119632981;\ p2 := p4$

$p0 := 2$

$p1 := 2.119632981$

$p2 := 2.094317350$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := -1$

$f1 := 0.283914304$

$f2 := -0.002612900$

$> c := f2$

$c := -0.002612900$

Comment

Step 10 of 93

Continuation of the above

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 6.213950202$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 11.16088318$

$> p5 := p4 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 := 2.094551432$

Continuation of the above

$> p0 := 2.119632981;\ p1 := 2.094317350;\ p2 := p5$

$p0 := 2.119632981$

$p1 := 2.094317350$

$p2 := 2.094551432$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := 0.283914304$

$f1 := -0.002612900$

$f2 := -5.53\ 10^{-7}$

$> c := f2$

$c := -5.53\ 10^{-7}$

Comment

Step 11 of 93

Continuation of the above

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 6.308539491$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 11.16144202$

$> p6 := p5 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}$

$p6 := 2.094551482$

Result: Thus, the required approximate real zero is $p_6 = \boxed{2.094551482}$

Comment

Step 12 of 93

b. Find the approximations to within 10^{-4} to all the real zeros of the polynomial $f(x)=x^3+3x^2-1$ Using Muller's method by the steps as shown below.

Step1: Find the interval $[a,b]$ in which the required real zero lies by using Maple Technology as shown below.

$> f := x^3 + 3\cdot x^2 - 1$

$f := x^3 + 3x^2 - 1$

$> a := \operatorname{convert}\left(f\big|_{x=0}, \mathit{float}, 8\right)$

$a := -1.$

$> b := \operatorname{convert}\left(f\big|_{x=1}, \mathit{float}, 8\right)$

$b := 3.$

Thus, the required interval is given by $[a,b]=[0,1]$

Comment

Step 13 of 93

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a,b]=[0,1]$ as shown below.

$> f := x \to x^3 + 3\cdot x^2 - 1$

$f := x \to x^3 + 3x^2 - 1$

$> p0 := -1;\ p1 := 1;\ p2 := 0$

$p0 := -1$

$p1 := 1$

$p2 := 0$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := 1$

$f1 := 3$

$f2 := -1$

Comment

Step 14 of 93

Continuation of the above

$> c := f2$

$c := -1$

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 3$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 1$

$> p3 := \operatorname{evalf}\left(p2 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := 0.4342585461$

Continuation of the above

$> p0 := p1;\ p1 := p2;\ p2 := p3$

$p0 := 1$

$p1 := 0$

$p2 := 0.4342585461$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := 3$

$f1 := -1$

$f2 := -0.3523658581$

$> c := f2$

$c := -0.3523658581$

Comment

Step 15 of 93

Continuation of the above

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 4.434258543$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 3.416970794$

$> p4 := p3 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := 0.5263702729$

Comment

Step 16 of 93

Continuation of the above

$> p0 := 1;\ p1 := p2;\ p2 := p4$

$p0 := 1$

$p1 := 0.4342585461$

$p2 := 0.5263702729$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := 3$

$f1 := -0.3523658581$

$f2 := -0.0229638781$

$> c := f2$

$c := -0.0229638781$

Comment

Step 17 of 93

Continuation of the above

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 4.960628822$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 4.033045481$

$> p5 := p4 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 := 0.5320248742$

Comment

Step 18 of 93

Continuation of the above

$> p0 := p1;\ p1 := p2;\ p2 := p5$

$p0 := 0.4342585461$

$p1 := 0.5263702729$

$p2 := 0.5320248742$

$> f0 := f(p0);\ f1 := f(p1);\ f2 := f(p2)$

$f0 := -0.3523658581$

$f1 := -0.0229638781$

$f2 := -0.0002587106$

$> c := f2$

$c := -0.0002587106$

Continuation of the above

$> a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 4.492653829$

$> b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 4.040747828$

$> p6 := p5 - \dfrac{2\cdot c}{b+\left(\dfrac{b}{\operatorname{abs}(b)}\right)\cdot \operatorname{sqrt}(b^2-4\cdot a\cdot c)}$

Step 19 of 93

...ontinuation of the above

> $p0 := p1; p1 := p2; p2 := p6$

$p0 :=$ [illegible]

$p1 :=$ [illegible]

$p2 :=$ [illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 :=$ [illegible]

$f1 :=$ [illegible]

$f2 :=$ [illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 20 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p7 := p6 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p7 :=$ [illegible]

Result: Thus, the required approximate real zero is $p_7 = \boxed{\text{[illegible]}}$

Comment

Step 21 of 93

Step3: Find the interval $[a_2, b_2]$ in which the required real zero lies by using Maple technology as shown below.

> $a2 := convert\left(f\Big|_{x=-0.8}, float, 8\right)$

$a2 := 0.408$

> $b2 := convert\left(f\Big|_{x=-0.5}, float, 8\right)$

$b2 := -0.375$

Thus, the required interval is given by $[a_2, b_2] = [-0.8, -0.5]$

Comment

Step 22 of 93

Step4: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a_2, b_2] = [-0.8, -0.5]$ as shown below.

> $f := x \to x^3 + 3\cdot x^2 - 1$

$f := x \to x^3 + 3x^2 - 1$

> $p0 := -2; p1 := -1; p2 := 0$

$p0 := -2$

$p1 := -1$

$p2 := 0$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 3$

$f1 := 1$

$f2 := -1$

Continuation of the above

> $c := f2$

$c := -1$

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 0$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := -2$

> $p3 := evalf\left(p2 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := -0.5000000000$

Comment

Step 23 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := -1$

$p1 := 0$

$p2 := -0.5000000000$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 1$

$f1 := -1$

$f2 := -0.3750000000$

> $c := f2$

$c := -0.3750000000$

Comment

Step 24 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 1.500000000$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := -2.000000000$

> $p4 := p3 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := -0.6666666667$

Comment

Step 25 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := 0$

$p1 := -0.5000000000$

$p2 := -0.6666666667$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -1$

$f1 := -0.3750000000$

$f2 := 0.037037038$

> $c := f2$

$c := 0.037037038$

Comment

Step 26 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 1.833333340$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := -2.777777784$

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 := -0.6532138881$

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

$p0 := -0.5000000000$

$p1 := -0.6666666667$

$p2 := -0.6532138881$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.3750000000$

$f1 := 0.037037038$

$f2 := 0.001346373$

> $c := f2$

$c := 0.001346373$

Comment

Step 27 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 1.180119718$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := -2.637157069$

> $p6 := p5 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p6 := -0.6527032318$

Comment

Step 28 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p6$

$p0 := -0.6666666667$

$p1 := -0.6532138881$

$p2 := -0.6527032318$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 0.037037038$

$f1 := 0.001346373$

$f2 := -0.0000010896$

> $c := f2$

$c := -0.0000010896$

Comment

Step 29 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 1.027330102$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := -2.638163287$

> $p7 := p6 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p7 := -0.6527036448$

Result: Thus, the required approximate real zero is $p_7 = \boxed{-0.6527036448}$

Comment

Step 30 of 93

Step5: Find the interval in which the required real zero lies by using Maple technology as shown below.

> $a3 := convert\left(f\Big|_{x=-3}, float, 8\right)$

$a3 := -1.$

> $b3 := convert\left(f\Big|_{x=-2}, float, 8\right)$

$b3 := 3.$

Thus, the required interval is given by $[a_3, b_3] = [-3, -2]$

Step6: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a_3, b_3] = [-3, -2]$ as shown below.

> $f := x \to x^3 + 3\cdot x^2 - 1$

$f := x \to x^3 + 3x^2 - 1$

> $p0 := -5; p1 := -4; p2 := -3$

$p0 := -5$

$p1 := -4$

$p2 := -3$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -51$

$f1 := -17$

$f2 := -1$

Comment

Step 31 of 93

Continuation of the above

> $c := f2$

$c := -1$

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := -9$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 7$

> $p3 := evalf\left(p2 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := -2.811419515$

Comment

Step 32 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := -4$

$p1 := -3$

$p2 := -2.811419515$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -17$

$f1 := -1$

$f2 := 0.49055518$

> $c := f2$

$c := 0.49055518$

Comment

Step 33 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := -6.811419522$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 6.619578882$

> $p4 := p3 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := -2.880601383$

Comment

Step 34 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := -3$

$p1 := -2.811419515$

$p2 := -2.880601383$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -1$

$f1 := 0.49055518$

$f2 := -0.00924648$

> $c := f2$

$c := -0.00924648$

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := -5.692020418$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 7.618244926$

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 := -2.879386552$

Comment

Step 35 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

$p0 := -2.811419515$

$p1 := -2.880601383$

$p2 := -2.879386552$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 0.49055518$

$f1 := -0.00924648$

$f2 := -0.00000995$

> $c := f2$

$c := -0.00000995$

Comment

Step 36 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := -5.571517629$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 7.596371408$

> $p6 := p5 - \frac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p6 := -2.879385242$

Result: Thus, the required approximate real zero is $p_6 = \boxed{-2.879385242}$

Comment

Step 37 of 93

c. Find the approximations to within 10^{-4} to all the real zeros of the polynomial $f(x) = x^3 - x - 1$ Using Muller's method by the steps as shown below.

Step1: Find the interval $[a, b]$ in which the required real zero lies by using Maple Technology as shown below.

> $f := x^3 - x - 1$

$f := x^3 - x - 1$

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Thus, the required interval is given by $[1,2]$

Comment

Step 38 of 93

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology as shown below.

> $f := x \to x^3 - x - 1$

$f := x \to x^3 - x - 1$

> $p0 := 0; p1 := 1; p2 := 2$

$p0 := 0$

$p1 := 1$

$p2 := 2$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -1$

$f1 := -1$

$f2 := 5$

Continuation of the above

> $c := f2$

$c := 5$

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p3 := evalf\left(p2 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}\right)$

$p3 :=$ [illegible]

Comment

Step 39 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := 1$

$p1 := 2$

$p2 :=$ [illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -1$

$f1 := 5$

$f2 :=$ [illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 40 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p4 := p3 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p4 :=$ [illegible]

Comment

Step 41 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := 2$

$p1 :=$ [illegible]

$p2 :=$ [illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 5$

$f1 :=$ [illegible]

$f2 :=$ [illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 42 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p5 :=$ [illegible]

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

$p0 :=$ [illegible]

$p1 :=$ [illegible]

$p2 :=$ [illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 :=$ [illegible]

$f1 :=$ [illegible]

$f2 :=$ [illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 43 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p6 := p5 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p6 :=$ [illegible]

Comment

Step 44 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p6$

$p0 := 1.321742825$

$p1 := 1.324689527$

$p2 := 1.324717958$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.012652697$

$f1 := -0.000121241$

$f2 := 3.\ 10^{-9}$

> $c := f2$

$c := 3.\ 10^{-9}$

Comment

Step 45 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 3.964344135$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 4.264612726$

> $p7 := p6 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p7 := 1.324717957$

Result: Thus, the required approximate real zero is $p_7 = \boxed{1.324717957}$

Comment

Step 46 of 93

d. Find the approximations to within 10^{-4} to all the real zeros of the polynomial $f(x) = x^4 + 2x^2 - x - 3$ Using Muller's method by the steps as shown below.

Step1: Find the interval $[a_1, b_1]$ in which the required real zero lies by using Maple technology as shown below.

> $f := x^4 + 2\cdot x^2 - x - 3$

$f := x^4 + 2x^2 - x - 3$

> $a1 := convert\left(f\Big|_{x=1}, float, 8\right)$

$a1 := -1.$

> $b1 := convert\left(f\Big|_{x=2}, float, 8\right)$

$b1 := 19.$

Thus, the required interval is given by $[a_1, b_1] = [1,2]$

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a_1, b_1] = [1,2]$ as shown below.

> $f := x \to x^4 + 2\cdot x^2 - x - 3$

$f := x \to x^4 + 2x^2 - x - 3$

> $p0 := 0; p1 := 1; p2 := 2$

$p0 := 0$

$p1 := 1$

$p2 := 2$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -3$

$f1 := -1$

$f2 := 19$

Comment

Step 47 of 93

Continuation of the above

> $c := f2$

$c := 19$

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 9$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 29$

> $p3 := evalf\left(p2 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}\right)$

$p3 := 1.084998005$

Comment

Step 48 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := 1$

$p1 := 2$

$p2 := 1.084998005$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -1$

$f1 := 19$

$f2 := -0.344708155$

> $c := f2$

$c := -0.344708155$

Comment

Step 49 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 13.43221469$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 8.851208222$

> $p4 := p3 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p4 := 1.121878612$

Comment

Step 50 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := 2$

$p1 := 1.084998005$

$p2 := 1.121878612$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 19$

$f1 := -0.344708155$

$f2 := -0.020552162$

> $c := f2$

$c := -0.020552162$

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 14.06682160$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 9.308127459$

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p5 := 1.124079273$

Comment

Step 51 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

$p0 := 1.084998005$

$p1 := 1.121878612$

$p2 := 1.124079273$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.344708155$

$f1 := -0.020552162$

$f2 := -0.000401602$

> $c := f2$

$c := -0.000401602$

Comment

Step 52 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 9.397335147$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 9.177274661$

> $p6 := p5 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p6 := 1.124123032$

Comment

Step 53 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p6$

$p0 := 1.121878612$

$p1 := 1.124079273$

$p2 := 1.124123032$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.020552162$

$f1 := -0.000401602$

$f2 := 2.1\ 10^{-8}$

> $c := f2$

$c := 2.1\ 10^{-8}$

Comment

Step 54 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 9.566797926$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 9.178484860$

> $p7 := p6 - \frac{2\cdot c}{b+\left(\frac{b}{abs(b)}\right)\cdot sqrt(b^2-4\cdot a\cdot c)}$

$p7 := 1.124123030$

Result: Thus, the required approximate real zero is $p_7 = \boxed{1.124123030}$

Step3: Find the interval $[a_2, b_2]$ in which the required real zero lies by using Maple technology as shown below.

> $a2 := convert\left(f\Big|_{x=-1}, float, 8\right)$

$a2 := 1.$

> $b2 := convert\left(f\Big|_{x=0}, float, 8\right)$

$b2 := -3.$

Thus, the required interval is given by $[a_2, b_2] = [-1,0]$

Comment

Step 55 of 93

Step4: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a_2, b_2] = [-1,0]$ as shown below.

> $f := x \to x^4 + 2\cdot x^2 - x - 3$

$f := x \to x^4 + 2x^2 - x - 3$

> $p0 := -2; p1 := -1; p2 := -0.5$

[illegible]

Comment

Step 56 of 93

Continuation of the above

> $c := f2$

$c :=$ [illegible]

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p3 := evalf\left(p2 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 :=$ [illegible]

Comment

Step 57 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

[illegible]

Comment

Step 58 of 93

Continuation of the above

[illegible]

Comment

Step 59 of 93

Continuation of the above

[illegible]

Comment

Step 60 of 93

Continuation of the above

[illegible]

Comment

Step 61 of 93

Continuation of the above

[illegible]

Result: Thus, the required approximate real zero is [illegible]

Comment

Step 62 of 93

e. Find the approximations to within 10^{-4} to all the real zeros of the polynomial $f(x)=x^3+4.001x^2+4.002x+1.101$ Using Muller's method by the steps as shown below.

Step1: Find the interval $[a_1,b_1]$ in which the required real zero lies by using Maple technology as shown below.

> $f := x^3+4.001\cdot x^2+4.002\cdot x+1.101$

$f := x^3+4.001x^2+4.002x+1.101$

> $a1 := convert\left(f\big|_{x=-1}, float, 8\right)$

$a1 := 0.100$

> $b1 := convert\left(f\big|_{x=-0.5}, float, 8\right)$

$b1 := -0.02475$

Thus, the required interval is given by $[a_1,b_1]=[-1,-0.5]$

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a_1,b_1]=[-1,-0.5]$ as shown below.

> $f := x \rightarrow x^3+4.001\cdot x^2+4.002\cdot x+1.101$

$f := x \rightarrow x^3+4.001x^2+4.002x+1.101$

> $p0 := -1; p1 := -0.5; p2 := 0$

$p0 := -1$

$p1 := -0.5$

$p2 := 0$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 0.100$

$f1 := -0.02475$

$f2 := 1.101$

Comment

Step 63 of 93

Continuation of the above

> $c := f2$

$c := 1.101$

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 2.501000000$

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 3.502000000$

> $p3 := evalf\left(p2 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := -0.4766383268$

Comment

Step 64 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := -0.5$

$p1 := 0$

$p2 := -0.4766383268$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.02475$

$f1 := 1.101$

$f2 := -0.005827668$

> $c := f2$

$c := -0.005827668$

Comment

Step 65 of 93

Continuation of the above

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 3.024361658$

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 0.8806274686$

> $p4 := p3 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := -0.4701646240$

Comment

Step 66 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := 0$

$p1 := -0.4766383268$

$p2 := -0.4701646240$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 1.101$

$f1 := -0.005827668$

$f2 := -0.000090810$

> $c := f2$

$c := -0.000090810$

Continuation of the above

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 3.054196984$

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 0.9059507359$

> $p5 := p4 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 := -0.4700644206$

Comment

Step 67 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

$p0 := -0.4766383268$

$p1 := -0.4701646240$

$p2 := -0.4700644206$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.005827668$

$f1 := -0.000090810$

$f2 := -3.10\ 10^{-7}$

> $c := f2$

$c := -3.10\ 10^{-7}$

Comment

Step 68 of 93

Continuation of the above

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 2.583577201$

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 0.9034218497$

> $p6 := p5 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p6 := -0.4700640775$

Result: Thus, the required approximate real zero is $p_6 = \boxed{-0.4700640775}$

Comment

Step 69 of 93

Step3: Find the interval $[a_2,b_2]$ in which the required real zero lies by using Maple technology as shown below.

> $a2 := convert\left(f\big|_{x=-3}, float, 8\right)$

$a2 := -1.896$

> $b2 := convert\left(f\big|_{x=-2}, float, 8\right)$

$b2 := 1.101$

Thus, the required interval is given by $[a_2,b_2]=[-3,-2]$

Comment

Step 70 of 93

Step4: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a_2,b_2]=[-3,-2]$ as shown below.

> $f := x \rightarrow x^3+4.001\cdot x^2+4.002\cdot x+1.101$

$f := x \rightarrow x^3+4.001x^2+4.002x+1.101$

> $p0 := -3.5; p1 := -3; p2 := -2.8$

$p0 := -3.5$

$p1 := -3$

$p2 := -2.8$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -6.76875$

$f1 := -1.896$

$f2 := -0.68876$

Continuation of the above

> $c := f2$

$c := -0.68876$

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := -5.299000000$

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 4.976400000$

> $p3 := evalf\left(p2 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := -2.631284564$

Comment

Step 71 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := -3$

$p1 := -2.8$

$p2 := -2.631284564$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -1.896$

$f1 := -0.68876$

$f2 := 0.05404104$

> $c := f2$

$c := 0.05404104$

Comment

Step 72 of 93

Continuation of the above

> $a := \dfrac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := -4.430284514$

> $b := \dfrac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 3.655228330$

> $p4 := p3 - \dfrac{2\cdot c}{b+\left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := -2.645813308$

Comment

Step 73 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := -2.8$

$p1 := -2.631284564$

$p2 := -2.645813308$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.68876$

$f1 := 0.05404104$

$f2 := -0.00079343$

Step 74 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 :=$ [illegible]

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

[illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

[illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 75 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p6 := p5 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p6 :=$ [illegible]

Result: Thus, the required approximate real zero is $p_6 = \boxed{\text{[illegible]}}$

Comment

Step 76 of 93

Step1: Find the interval in which the required real zero lies by using Maple technology as shown below.

> $a := \text{convert}(f|_{x=\text{[illegible]}}, \text{float}, 8)$

$a :=$ [illegible]

> $b := \text{convert}(f|_{x=\text{[illegible]}}, \text{float}, 8)$

$b :=$ [illegible]

Thus, the required interval is given by [illegible]

Comment

Step 77 of 93

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval [illegible] as shown below.

[illegible]

Comment

Step 78 of 93

Continuation of the above

> $c := f2$

$c :=$ [illegible]

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p3 := \text{evalf}\left(p2 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 :=$ [illegible]

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

[illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

[illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 79 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p4 := p3 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 :=$ [illegible]

Comment

Step 80 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

[illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

[illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 81 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p5 :=$ [illegible]

Comment

Step 82 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

[illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

[illegible]

> $c := f2$

$c :=$ [illegible]

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a :=$ [illegible]

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b :=$ [illegible]

> $p6 := p5 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p6 :=$ [illegible]

Comment

Step 83 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p6$

[illegible]

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

[illegible]

> $c := f2$

$c :=$ [illegible]

Comment

Step 84 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 2.683205902$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 0.9034886679$

> $p7 := p6 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p7 := -0.4700640784$

Comment

Step 85 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p7$

$p0 := -0.4700783570$

$p1 := -0.4700640773$

$p2 := -0.4700640784$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.000012900$

$f1 := 1.10^{-9}$

$f2 := -1.10^{-9}$

> $c := f2$

$c := -1.10^{-9}$

Comment

Step 86 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 64063.10598$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 1.818111350$

> $p8 := p7 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p8 := -0.4700640778$

Result: Thus, the required approximate real zero is $p_8 = \boxed{-0.4700640778}$

f. Find the approximations to within 10^{-4} to all the real zeros of the polynomial $f(x) = x^5 - x^4 + 2x^3 - 3x^2 + x - 4$ Using Muller's method by the steps as shown below.

Step1: Find the interval $[a,b]$ in which the required real zero lies by using Maple Technology as shown below.

> $a := \text{convert}(f|_{x=1}, \text{float}, 8)$

$a := -4.$

> $b := \text{convert}(f|_{x=2}, \text{float}, 8)$

$b := 18.$

Thus, the required interval is given by $[1,2]$

Comment

Step 87 of 93

Step2: Find the approximation to within 10^{-4} using Muller's method by Applying Maple technology in the interval $[a,b]=[1,2]$ as shown below.

> $f := x \rightarrow x^5 - x^4 + 2\cdot x^3 - 3\cdot x^2 + x - 4$

$f := x \rightarrow x^5 - x^4 + 2x^3 - 3x^2 + x - 4$

> $p0 := 0.5; p1 := 1; p2 := 1.5$

$p0 := 0.5$

$p1 := 1$

$p2 := 1.5$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -4.03125$

$f1 := -4$

$f2 := 0.03125$

Comment

Step 88 of 93

Continuation of the above

> $c := f2$

$c := 0.03125$

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 8.000000000$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 12.06250000$

> $p3 := \text{evalf}\left(p2 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}\right)$

$p3 := 1.497404860$

Comment

Step 89 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p3$

$p0 := 1$

$p1 := 1.5$

$p2 := 1.497404860$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -4$

$f1 := 0.03125$

$f2 := -0.013501897$

> $c := f2$

$c := -0.013501897$

Comment

Step 90 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 18.45981566$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 17.19659622$

> $p4 := p3 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

$p4 := 1.498189349$

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p4$

$p0 := 1.5$

$p1 := 1.497404860$

$p2 := 1.498189349$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := 0.03125$

$f1 := -0.013501897$

$f2 := -0.000010944$

> $c := f2$

$c := -0.000010944$

Comment

Step 91 of 93

Continuation of the above

> $a := \frac{((p1-p2)\cdot(f0-f2)-(p0-p2)\cdot(f1-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$a := 26.16743034$

> $b := \frac{((p0-p2)^2\cdot(f1-f2)-(p1-p2)^2\cdot(f0-f2))}{(p0-p2)\cdot(p1-p2)\cdot(p0-p1)}$

$b := 17.21765001$

> $p5 := p4 - \frac{2\cdot c}{b+\left(\frac{b}{\text{abs}(b)}\right)\cdot\text{sqrt}(b^2-4\cdot a\cdot c)}$

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Step 92 of 93

Continuation of the above

> $p0 := p1; p1 := p2; p2 := p5$

$p0 := 1.497404860$

$p1 := 1.498189349$

$p2 := 1.498189985$

> $f0 := f(p0); f1 := f(p1); f2 := f(p2)$

$f0 := -0.013501897$

$f1 := -0.000010944$

$f2 := 6.\ 10^{-9}$

> $c := f2$

$c := 6.\ 10^{-9}$

Comment

Step 93 of 93

Continuation of the above

> $a := \frac{((p1 - p2)\cdot(f0 - f2) - (p0 - p2)\cdot(f1 - f2))}{(p0 - p2)\cdot(p1 - p2)\cdot(p0 - p1)}$

$a := 25.29429569$

> $b := \frac{((p0 - p2)^2\cdot(f1 - f2) - (p1 - p2)^2\cdot(f0 - f2))}{(p0 - p2)\cdot(p1 - p2)\cdot(p0 - p1)}$

$b := 17.21699722$

> $p6 := p5 - \frac{2\cdot c}{b + \left(\frac{b}{\mathrm{abs}(b)}\right)\cdot \mathrm{sqrt}(b^2 - 4\cdot a\cdot c)}$

$p6 := 1.498189985$

Result: Thus, the required approximate real zero is $p_6 = \boxed{1.498189985}$

Comment